§8.10b Manipulation of Known Taylor Series

Deriving Taylor Series from a Basic List

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.
Taylor Polynomials and Infinite Series by B. Goldstein. © 2015 Benjamin Goldstein.

Learning Goals: Students will be able to…

- Find a Taylor or Maclaurin series for a function.
- Use a basic list of Taylor series to find other Taylor series.

Deriving Taylor Series from a Basic List

The list below provides the power series for several elementary functions with the corresponding intervals of convergence.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots \qquad -\infty < x < \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots \qquad -\infty < x < \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots \qquad -\infty < x < \infty$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots + x^n + \cdots \qquad -1 < x < 1$$

Deriving Taylor Series from a Basic List

The direct determination of Taylor or Maclaurin coefficients using successive differentiation can be difficult.

There exists a shortcut for finding the coefficients indirectly—using the coefficients of a known Taylor or Maclaurin series.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $f(x) = \cos(x^2)$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $g(x) = \frac{d}{dx}\left[\cos(x^2)\right]$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $f(x)=\dfrac{x}{1-x^2}$. Include the first four nonzero terms and the general term.

Example: Deriving Taylor Series from a Basic List

Find the Maclaurin series for the function $h(x)=\int_0^x e^{-t^2}\,dt$. Include the first four nonzero terms and the general term. Then approximate the value of $h(1)$ with an error of less than $1/40$.

Example: Deriving Taylor Series from a Basic List

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