§8.8a Alternating Series

Alternating Series

Alternating Series Remainder

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Taylor Polynomials and Infinite Series by B. Goldstein. © 2015 Benjamin Goldstein.

Learning Goals: Students will be able to…

- Use the Alternating Series Test to determine whether an infinite series converges.
- Determine the Alternating Series Remainder of an alternating series.
- Use the Alternating Series Remainder to approximate the sum of an alternating series.

Alternating Series

So far, most series we have dealt with have had positive terms. In this section, we will study series that contain both positive and negative terms.

The simplest such series is an **alternating series**, whose terms alternate in sign.

For example, the geometric series $\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n = \sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^n = 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\cdots$ is an *alternating geometric series* with $r=-1/2$.

Alternating series occur in two ways: either the odd terms are negative or the even terms are negative.

Alternating Series

THEOREM ALTERNATING SERIES TEST

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty}(-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

converge if the following two conditions are met.

1. $\lim_{n\to\infty} a_n = 0$ **2.** $a_{n+1} \le a_n$, for all n

The second condition in the Alternating Series Test can be modified to require only that $a_{n+1} \le a_n$ for all n greater than or equal to some integer N.

Example: Alternating Series

Determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$.

Example: Alternating Series

Determine the convergence or divergence of the series $\sum_{n=2}^{\infty}\frac{\cos(n\pi)n}{n-1}$.

Alternating Series Remainder

For a convergent alternating series, the partial sum S_N can be a useful approximation for the sum S of the series. The error involved in using $S \approx S_N$ is the remainder $R_N = S - S_N$.

THEOREM ALTERNATING SERIES REMAINDER

If a convergent alternating series satisfies the condition $a_{n+1} \leq a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is less than (or equal to) the first neglected term. That is,

$$|S - S_N| = |R_N| \leq a_{N+1}.$$

Example: Alternating Series Remainder

Approximate the sum of the convergent alternating series $\sum_{n=0}^{\infty} \frac{(-1)^n 0.5^{2n+1}}{(2n+1)!}$ by using the first two terms. Estimate the error of this approximation.

Example: Alternating Series Remainder

Approximate the sum of the convergent alternating series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ by using the first four terms. Estimate the error of this approximation, and give bounds for the value of the series.