

§8.7a The Integral Test and p-Series

The Integral Test

p-Series and Harmonic Series

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.
Taylor Polynomials and Infinite Series by B. Goldstein. © 2015 Benjamin Goldstein.

Learning Goals: Students will be able to...

- Use the Integral Test to determine whether an infinite series converges or diverges.
- Use properties of p-series and harmonic series.

The Integral Test

THEOREM THE INTEGRAL TEST

If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x) dx$$

either both converge or both diverge.

The convergence or divergence of $\sum a_n$ is not affected by deleting the first N terms.

Similarly, when the conditions for the Integral Test are satisfied for all $x \geq N > 1$, we can simply use the integral $\int_N^{\infty} f(x) dx$ to test for convergence or divergence.

Example: The Integral Test

Determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{2n}{n^2+1}$.

Example: The Integral Test

Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

p-Series and Harmonic Series

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ is a **p-series**, where p is a positive constant.

For $p=1$, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots$ is the **harmonic series**.

Do not confuse p-series with geometric series. Note the differences with these two examples:

p-series:
$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

Geometric series:
$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots$$

p -Series and Harmonic Series**THEOREM** CONVERGENCE OF p -SERIESThe p -series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

1. converges if $p > 1$, and
2. diverges if $0 < p \leq 1$.

The harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n}$, is a divergent p -series with $p = 1 \leq 1$.Example: p -Series and Harmonic SeriesDetermine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt[4]{n}}{n}$ and $\sum_{n=1}^{\infty} n^{-4/3}$.