





Example: The Ratio Test

Determine whether the series  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$  converges or diverges.

The Ratio Test

To determine the radius of convergence of a power series with general term  $u_n$ , use the Ratio Test.

- If  $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \infty$ , the series converges only at its center, so its radius of convergence is  $R = 0$ .
- If  $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = 0$ , the series converges for all  $x$ , so its radius of convergence is  $R = \infty$ .
- Otherwise, set  $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| < 1$ . Rewrite this inequality in the format  $|x - c| < R$ , where  $R$  is the series' radius of convergence.

Example: The Ratio Test

Find the radius of convergence of  $\sum_{n=0}^{\infty} n! x^n$ .

Example: The Ratio Test

Find the radius of convergence of  $\sum_{n=1}^{\infty} \frac{n \cdot x^n}{4^n}$ .

Example: The Ratio Test

Find the radius of convergence of  $\sum_{n=0}^{\infty} \frac{(x-1)^{3n}}{(3n)!}$ .