§8.2 An Introduction to Taylor Polynomials

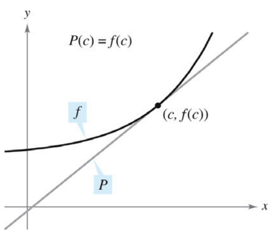
Polynomial Approximations of Elementary Functions

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.
Taylor Polynomials and Infinite Series by B. Goldstein. © 2015 Benjamin Goldstein.

Learning Goals: Students will be able to…

- Find polynomial approximations of elementary functions and compare them with the elementary functions.

Polynomial Approximations of Elementary Functions



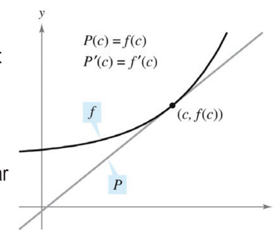
The goal of this (and the next) section is to show how polynomial functions can be used as approximations for other elementary functions.

To find a polynomial function P that approximates another function f, begin by choosing a number c in the domain of f at which f and P have the same value. That is, $P(c) = f(c)$.

The approximating polynomial is said to be **expanded about *c*** or **centered at *c***. Geometrically, the requirement that $P(c) = f(c)$ means that the graph of P passes through the point $(c, f(c))$.

Of course, there are many polynomials whose graphs pass through the point $(c, f(c))$. Our task is to find a polynomial whose graph resembles the graph of f near this point.

Polynomial Approximations of Elementary Functions



One way to do this is to impose the additional requirement that the slope of the polynomial function be the same as the slope of the graph of f at the point $(c, f(c))$. That is, $P'(c) = f'(c)$.

With these two requirements, we can obtain a simple linear approximation of f.

Example: Polynomial Approximations of Elementary Functions

Given $f(x) = e^x$, find a first-degree polynomial function $P_1(x) = a_0 + a_1 x$ whose value and slope agree with the value and slope of f at $x = 0$.

Example: Polynomial Approximations of Elementary Functions

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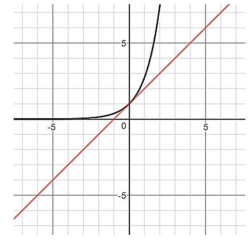
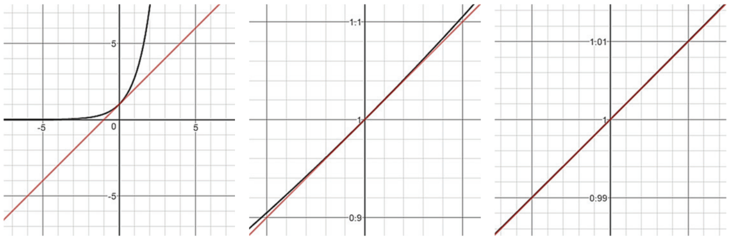


Polynomial Approximations of Elementary Functions

In the previous example, we saw that, at points near (0, 1), the graph of the first-degree polynomial function $P_1(x) = 1 + x$ is reasonably close to the graph of $f(x) = e^x$.

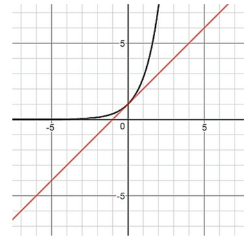
As we move closer to (0, 1), the graph of $P_1(x)$ more closely resembles the graph of $f(x) = e^x$, and the accuracy of the approximation increases.



Polynomial Approximations of Elementary Functions

As we move away from (0, 1), however, the graphs move farther and farther from each other and the accuracy of the approximation decreases.

To improve the approximation, we can impose yet another requirement—that the values of the second derivatives of P and f agree when $x = 0$. That is, $P''(0) = f''(0)$.

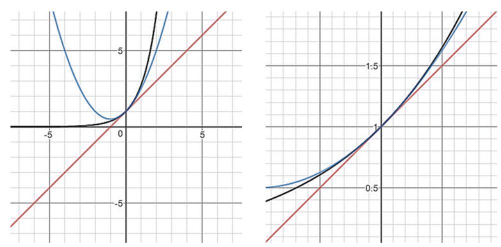


Example: Polynomial Approximations of Elementary Functions

Given $f(x) = e^x$, find a second-degree polynomial function $P_2(x) = a_0 + a_1x + a_2x^2$ such that $P_2(0) = f(0)$, $P_2'(0) = f'(0)$, and $P_2''(0) = f''(0)$.

Example: Polynomial Approximations of Elementary Functions

Given $f(x) = e^x$, find a second-degree polynomial function $P_2(x) = a_0 + a_1x + a_2x^2$ such that $P_2(0) = f(0)$, $P_2'(0) = f'(0)$, and $P_2''(0) = f''(0)$.



Example: Polynomial Approximations of Elementary Functions

Given $f(x) = e^x$, find a third-degree polynomial function $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ such that $P_3(0) = f(0)$, $P_3'(0) = f'(0)$, $P_3''(0) = f''(0)$, and $P_3'''(0) = f'''(0)$.

Example: Polynomial Approximations of Elementary Functions

Given $f(x) = e^x$, find a third-degree polynomial function $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ such that $P_3(0) = f(0)$, $P_3'(0) = f'(0)$, $P_3''(0) = f''(0)$, and $P_3'''(0) = f'''(0)$.