Improper Integrals with Infinite Discontinuities

The second type of improper integral is one that has an infinite discontinuity *at or between* the limits of integration.

Note that a function f is said to have an **infinite discontinuity** at c when, *from the right or left*, $\lim_{x\to c} f(x) = \infty$ or $\lim_{x\to c} f(x) = -\infty$.

Improper Integrals with Infinite Discontinuities

DEFINITION OF IMPROPER INTEGRALS WITH INFINITE DISCONTINUITIES

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,dx = \lim_{c\to b^-} \int_a^c f(x)\,dx.$$

2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,dx = \lim_{c\to a^+} \int_c^b f(x)\,dx.$$

3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

In the first two cases, the improper integral **converges** if the limit exists—otherwise, the improper integral **diverges.** In the third case, the improper integral on the left diverges if either of the improper integrals on the right diverges.

Example: Improper Integrals with Infinite Discontinuities

Evaluate the improper integral $\int_0^5 \frac{10}{x}\,dx$ if it converges, or show that it diverges.

Example: Improper Integrals with Infinite Discontinuities

Evaluate the improper integral $\int_{-1}^1 \frac{1}{x^{2/3}}\,dx$ if it converges, or show that it diverges.