§7.2 Integration by Parts

Integration by Parts

Using the Tabular Method

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Find an antiderivative using integration by parts.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.
2.1D Determine higher order derivatives.
3.1A Recognize antiderivatives of basic functions.
3.3B Calculate antiderivatives, and evaluate definite integrals.

Integration by Parts

In this section, we will study an important integration technique called **integration by parts**. This technique can be applied to a wide variety of functions and is particularly useful for integrands involving *products* of algebraic and transcendental functions.

Integration by parts is based on the formula for the derivative of a product:

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

When u' and v' are continuous, we can integrate both sides of this equation to obtain

$$uv = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx = \int u\,dv + \int v\,du$$

By rewriting this equation, we obtain the next theorem.

Integration by Parts

THEOREM INTEGRATION BY PARTS

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du.$$

This formula expresses the original integral in terms of another integral. Depending on the choices of u and dv, it may be easier to evaluate the second integral than the original one.

Because the choices of u and dv are critical in the integration by parts process, the following guidelines are provided.

Integration by Parts

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Note that dv always includes the dx of the original integrand.

When using integration by parts, note that we can first choose dv or first choose u. After we choose, however, the choice of the other factor is determined—it must be the remaining portion of the integrand.

Also note that dv must contain the differential dx of the original integral.

Integration by Parts

SUMMARY OF COMMON INTEGRALS USING INTEGRATION BY PARTS

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \quad \int x^n \sin ax\,dx, \quad \text{or} \quad \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}\,dx$, $\sin ax\,dx$, or $\cos ax\,dx$.

2. For integrals of the form

$$\int x^n \ln x\,dx, \quad \int x^n \arcsin ax\,dx, \quad \text{or} \quad \int x^n \arctan ax\,dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\,dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \quad \text{or} \quad \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\,dx$.

Integration by Parts

We can use the acronym **LIATE** as a guideline for choosing u in integration by parts.

In order, check the integrand for the following:

- Is there a **Logarithmic** part?
- Is there an **Inverse trigonometric** part?
- Is there an **Algebraic** part?
- Is there a **Trigonometric** part?
- Is there an **Exponential** part?

Example: Integration by Parts

Find the indefinite integral: $\int x \sin(x)\, dx$

Example: Integration by Parts

Find the indefinite integral: $\int \frac{4x}{e^x}\, dx$

Example: Integration by Parts

Evaluate the definite integral: $\int_0^1 \arcsin(x)\, dx$

Example: Integration by Parts

Evaluate the definite integral: $\int_0^1 x^2 e^x\, dx$

Using the Tabular Method

In problems involving repeated applications of integration by parts, a tabular method can help to organize the work. This method works well for integrals of the form

$$\int x^n \sin(ax)\, dx,\ \int x^n \cos(ax)\, dx, \text{ and } \int x^n e^{ax}\, dx.$$

This method also works for integrals of the form

$$\int e^{ax} \sin(bx)\, dx \text{ and } \int e^{ax} \cos(bx)\, dx.$$

Example: Using the Tabular Method

Evaluate the definite integral: $\int_0^1 x^2 e^x \, dx$

Example: Using the Tabular Method

Find the indefinite integral: $\int x^3 \cos(2x) \, dx$

Example: Using the Tabular Method

Find the indefinite integral: $\int e^{3x} \cos(4x) \, dx$