

§7.1 Basic Integration Rules

Fitting Integrands to Basic Integration Rules

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Review procedures for fitting an integrand to one of the basic integration rules.

Fitting Integrands to Basic Integration Rules

In this chapter, we will study several integration techniques that greatly expand the set of integrals to which the basic integration rules can be applied.

In this section, we will look at various techniques that can be used to rewrite integrals to fit a specific integration rule.

Fitting Integrands to Basic Integration Rules

- Expand (numerator). $(1 + e^x)^2 = 1 + 2e^x + e^{2x}$
- Separate numerator. $\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
- Complete the square. $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$
- Divide improper rational function. $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
- Add and subtract terms in numerator. $\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$
- Use trigonometric identities. $\cot^2(x) = \csc^2(x) - 1$
- Multiply and divide by Pythagorean conjugate. $\frac{1}{1+\sin(x)} = \frac{1}{1+\sin(x)} \cdot \frac{1-\sin(x)}{1-\sin(x)} = \frac{1-\sin(x)}{1-\sin^2(x)} = \frac{1-\sin(x)}{\cos^2(x)} = \sec^2(x) - \sec(x)\tan(x)$
- Multiply and divide by a common term. $\frac{1}{1+e^x} = \frac{1}{1+e^x} \cdot \frac{e^{-x}}{e^{-x}} = \frac{e^{-x}}{e^{-x}+1}$

Example: Expand

Find the indefinite integral: $\int x \left(1 + \frac{1}{x}\right)^3 dx$

Example: Complete the Square

Find the indefinite integral: $\int \frac{12}{\sqrt{9-8x-x^2}} dx$

Example: Divide Improper Rational Function

Find the indefinite integral: $\int \frac{4x}{x-8} dx$

Example: Manipulate Numerator

Find the indefinite integral: $\int \frac{2x}{x^2 + 4x + 4} dx$

Example: Use Trigonometric Identities

Find the indefinite integral: $\int \tan^2(2x) dx$

Example: Pythagorean Conjugate

Find the indefinite integral: $\int \frac{1}{\sec(x)-1} dx$

Example: Common Term

Find the indefinite integral: $\int \frac{5}{3e^x - 2} dx$