

**§6.2b Volume: Solids with Known Cross Sections**  
 Solids with Known Cross Sections

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.  
*Calculus, AP Edition, 9th ed.* by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

**Learning Objectives:** Students will be able to...

- Find the volume of a solid with known cross sections.

**Solids with Known Cross Sections**

With the disk method, we can find the volume of a solid having a circular cross section whose area is  $A = \pi R^2$ .

This method can be generalized to solids of any shape, as long as we know a formula for the area of an arbitrary cross section.

Some common cross sections are squares, rectangles, triangles, semicircles, and trapezoids.

**Solids with Known Cross Sections**

**VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS**

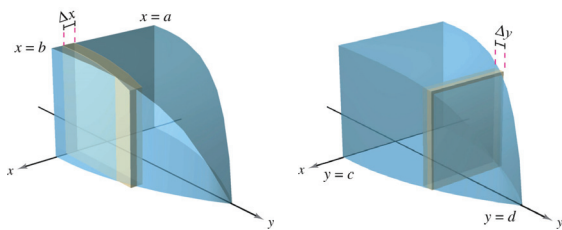
1. For cross sections of area  $A(x)$  taken perpendicular to the  $x$ -axis,

$$\text{Volume} = \int_a^b A(x) \, dx.$$

2. For cross sections of area  $A(y)$  taken perpendicular to the  $y$ -axis,

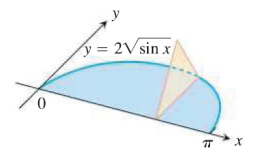
$$\text{Volume} = \int_c^d A(y) \, dy.$$

**Solids with Known Cross Sections**



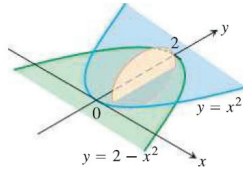
**Example: Solids with Known Cross Sections**

Find the volume of the solid whose bases are bounded by the graph of  $y = 2\sqrt{\sin(x)}$  and the  $x$ -axis, with equilateral triangle cross sections taken perpendicular to the  $x$ -axis.



Example: Solids with Known Cross Sections

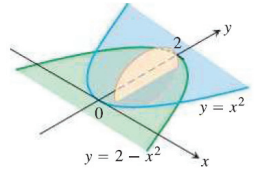
Find the volume of the solid whose bases are bounded by the graphs of  $y = x^2$  and  $y = 2 - x^2$ , with semicircular cross sections taken perpendicular to the  $x$ -axis.



Problem from *Thomas' Calculus, 12th ed.* by Weir, Haas, & Thomas. © 2010 Pearson Education, Inc.

Example: Solids with Known Cross Sections (cont.)

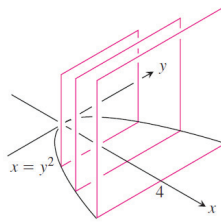
Find the volume of the solid whose bases are bounded by the graphs of  $y = x^2$  and  $y = 2 - x^2$ , with semicircular cross sections taken perpendicular to the  $x$ -axis.



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Example: Solids with Known Cross Sections

Find the volume of the solid whose bases are bounded by the graphs of  $x = y^2$  and  $x = 4$ , with square cross sections taken perpendicular to the  $x$ -axis.



Problem from *Calculus: Graphical, Numerical, Algebraic, 3rd ed.* by Finney, Demana, Waits, & Kennedy. © 2007 Pearson Education, Inc.

Example: Solids with Known Cross Sections

Find the volume of the solid whose bases are bounded by  $y = \ln(x)$ ,  $y = 4 - \frac{1}{2}x$ , and the  $x$ -axis, with rectangular cross sections of height  $y$  taken perpendicular to the  $y$ -axis.

Problem from *Calculus: Graphical, Numerical, Algebraic, 3rd ed.* by Finney, Demana, Waits, & Kennedy. © 2007 Pearson Education, Inc.