

§6.2b Volume: Solids with Known Cross Sections
Solids with Known Cross Sections

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Objectives: Students will be able to...

- Find the volume of a solid with known cross sections.

Learning Objectives: Students will be able to...

- 3.1A Recognize antiderivatives of basic functions.
- 3.2B Approximate a definite integral.
- 3.3B Calculate antiderivatives, and evaluate definite integrals.
- 3.4D Apply definite integrals to problems involving area, volume, (*BC: and length of a curve*).

Solids with Known Cross Sections

With the disk method, we can find the volume of a solid having a circular cross section whose area is $A = \pi R^2$.

This method can be generalized to solids of any shape, as long as we know a formula for the area of an arbitrary cross section.

Some common cross sections are squares, rectangles, triangles, semicircles, and trapezoids.

Solids with Known Cross Sections

VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

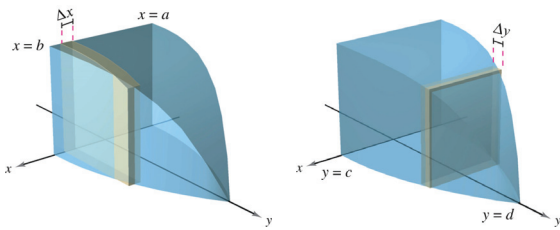
1. For cross sections of area $A(x)$ taken perpendicular to the x -axis,

$$\text{Volume} = \int_a^b A(x) dx.$$

2. For cross sections of area $A(y)$ taken perpendicular to the y -axis,

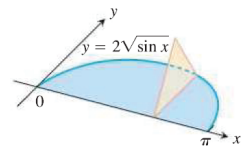
$$\text{Volume} = \int_c^d A(y) dy.$$

Solids with Known Cross Sections



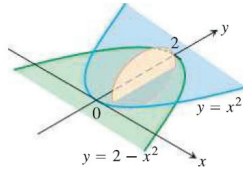
Example: Solids with Known Cross Sections

Find the volume of the solid whose bases are bounded by the graph of $y = 2\sqrt{\sin(x)}$ and the x -axis, with equilateral triangle cross sections taken perpendicular to the x -axis.



Example: Solids with Known Cross Sections

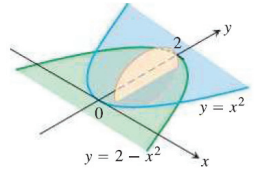
Find the volume of the solid whose bases are bounded by the graphs of $y = x^2$ and $y = 2 - x^2$, with semicircular cross sections taken perpendicular to the x -axis.



Problem from *Thomas' Calculus, 12th ed.* by Weir, Haas, & Thomas. © 2010 Pearson Education, Inc.

Example: Solids with Known Cross Sections (cont.)

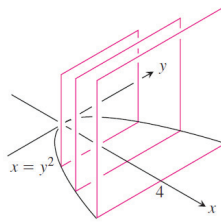
Find the volume of the solid whose bases are bounded by the graphs of $y = x^2$ and $y = 2 - x^2$, with semicircular cross sections taken perpendicular to the x -axis.



Problem from *Thomas' Calculus, 12th ed.* by Weir, Haas, & Thomas. © 2010 Pearson Education, Inc.

Example: Solids with Known Cross Sections

Find the volume of the solid whose bases are bounded by the graphs of $x = y^2$ and $x = 4$, with square cross sections taken perpendicular to the x -axis.



Problem from *Calculus: Graphical, Numerical, Algebraic, 3rd ed.* by Finney, Demana, Waits, & Kennedy. © 2007 Pearson Education, Inc.

Example: Solids with Known Cross Sections

Find the volume of the solid whose bases are bounded by $y = \ln(x)$, $y = 4 - \frac{1}{2}x$, and the x -axis, with rectangular cross sections of height y taken perpendicular to the y -axis.