§6.1 Area of a Region Between Two Curves

Area of a Region Between Two Curves

Area of a Region Between Intersecting Curves

Learning Goals: Students will be able to…

- Find the area of a region between two curves using integration.
- Find the area of a region between intersecting curves using integration.

Learning Objectives: Students will be able to…

3.1A Recognize antiderivatives of basic functions.

3.2B Approximate a definite integral.

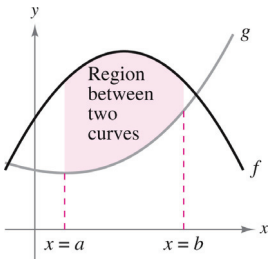
3.3B Calculate antiderivatives, and evaluate definite integrals.

3.4D Apply definite integrals to problems involving area, volume, *(BC: and length of a curve)*.

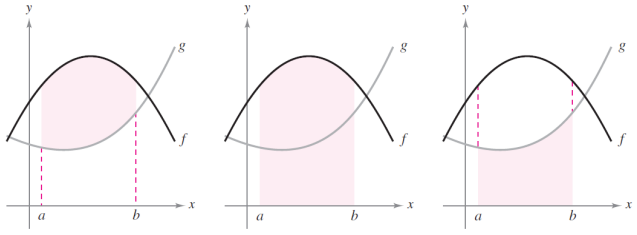
Area of a Region Between Two Curves

Consider two functions f and g that are continuous on the interval $[a, b]$. Also, the graphs of both f and g lie above the x-axis, and the graph of g lies below the graph of f.

We can geometrically interpret the area of the region between the graphs as the area of the region under the graph of g subtracted from the area of the region under the graph of f.



Area of a Region Between Two Curves



Area of region between f and g	=	Area of region under f	−	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	=	$\int_a^b f(x)\,dx$	−	$\int_a^b g(x)\,dx$

Area of a Region Between Two Curves

AREA OF A REGION BETWEEN TWO CURVES

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx.$$

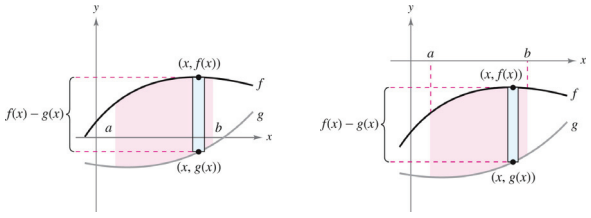
$f(x)$ is the "top" curve of the region, while $g(x)$ is the "bottom" curve of the region.

Thus, $A = \int_a^b [\text{top curve} - \text{bottom curve}]\,dx$.

Area of a Region Between Two Curves

In the previous example, the graphs of f and g are shown above the x-axis. This, however, is not necessary.

The same integrand $[f(x) - g(x)]$ can be used as long as f and g are continuous and $g(x) \le f(x)$ for all x in the interval $[a, b]$.



Area of a Region Between Two Curves

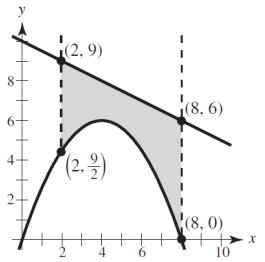
If $f(y)$ and $g(y)$ are continuous on $[a, b]$ and $g(y) \le f(y)$ for all y in $[a, b]$, then the area of the region bounded by the graphs of f and g and the horizontal lines $y = a$ and $y = b$ is $A = \int_a^b [f(y) - g(y)]\,dy$.

$f(y)$ is the "right" curve of the region, while $g(y)$ is the "left" curve of the region.

Thus, $A = \int_a^b [\text{right curve} - \text{left curve}]\,dy$.

Example: Area of a Region Between Two Curves

Find the area of the region bounded by the graphs of $f(x) = -\frac{3}{8}x^2 + 3x$, $g(x) = 10 - \frac{1}{2}x$, $x = 2$, and $x = 8$.



Area of a Region Between Intersecting Curves

In the previous example, the graphs do not intersect, and the values of a and b are given explicitly.

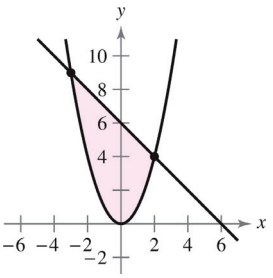
A more common problem involves the area of a region bounded by two *intersecting* graphs, where the values of a and b must be calculated.

For simpler problems, these intersection points can be determined algebraically. For more complicated problems, a graphing calculator may be required to determine the intersection points.

To find the area of the region between two curves that intersect at *more* than two points, determine all points of intersection. Then check to see which curve is above the other in each interval determined by these points, and evaluate the definite integrals separately.

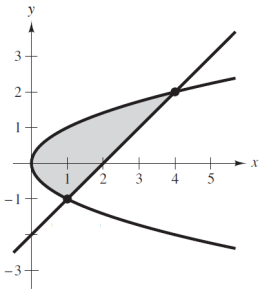
Example: Area of a Region Between Intersecting Curves

Find the area of the region bounded by the graphs of $f(x) = x^2$ and $g(x) = 6 - x$.



Example: Area of a Region Between Intersecting Curves

Find the area of the region bounded by the graphs of $f(y) = y^2$ and $g(y) = y + 2$.



Example: Area of a Region Between Intersecting Curves

Find the area of the region bounded by the graphs of $f(x) = \sqrt[3]{x}$ and $g(x) = -x^3 + 6x^2 - 9x + 4$.