

§5.3 Separation of Variables

Separation of Variables

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Recognize and solve differential equations that can be solved by separation of variables.
- Use differential equations to model and solve applied problems.

Separation of Variables

Consider a differential equation that can be written in the form $M(x) + N(y) \frac{dy}{dx} = 0$, where M is a continuous function of x alone and N is a continuous function of y alone.

For this type of equation, all x terms can be collected with dx and all y terms with dy , and a solution can be obtained via integration.

$$M(x) + N(y) \frac{dy}{dx} = 0$$

$$N(y) \frac{dy}{dx} = -M(x)$$

$$\int N(y) dy = \int -M(x) dx$$

Separation of Variables

Such equations are said to be **separable**, and the solution procedure is called **separation of variables**.

Original Differential Equation

$$\frac{dy}{dx} = \frac{x}{y^2 + 1}$$

$$x^2 + 3y \frac{dy}{dx} = 0$$

$$\sin(x) y' = \cos(x)$$

$$\frac{xy'}{e^y + 1} = 2$$

Rewritten with Variables Separated

$$(y^2 + 1) dy = x dx$$

$$3y dy = -x^2 dx$$

$$dy = \cot(x) dx$$

$$\frac{dy}{e^y + 1} = \frac{2}{x} dx$$

Separation of Variables

1. Perform separation of variables.
2. Perform antidifferentiation on each side.
3. Include constant of integration C on the side with the independent variable.
4. Use the initial condition to solve for C .
5. Solve for the dependent variable.
 - Rewrite any absolute values based on the initial condition.
 - Replace $|a|$ with a if $a \geq 0$ for the initial condition.
 - Replace $|a|$ with $-a$ if $a < 0$ for the initial condition.

Example: Separation of Variables

Given the differential equation $\sqrt{x} + \sqrt{y} \frac{dy}{dx} = 0$, find the particular solution that satisfies the initial condition $y(9) = 1$.

Example: Separation of Variables

Given the differential equation $xy' = y^2$ find the particular solution that satisfies the initial condition $y(1) = 2$.

Example: Separation of Variables

Given the differential equation $4xy + y' = 0$, find the particular solution that satisfies the initial condition $y(2) = -1$.

Example: Separation of Variables

Given the differential equation $yy' = 2\sin(4x)$, find the particular solution that satisfies the initial condition $y(0) = -2$.