§5.1b Euler's Method

Euler's Method

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

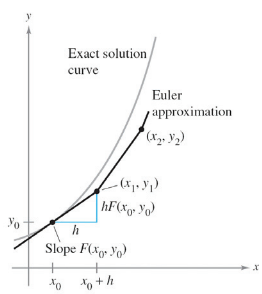
Learning Goals: Students will be able to…

- Use Euler's Method to approximate solutions of differential equations.

Euler's Method

Euler's Method is a numerical approach to approximating the particular solution of the differential equation $y' = F(x, y)$ that passes through the point (x_0, y_0).

From the given information, we know that the graph of the solution passes through the point (x_0, y_0) and has a slope of $F(x_0, y_0)$ at this point. This gives us a "starting point" for approximating the solution.

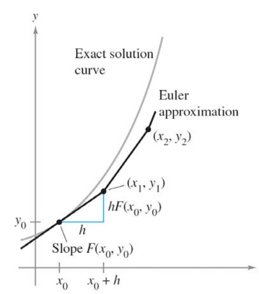


Euler's Method

From this starting point, we can proceed in the direction indicated by the slope. Using a small step h, move along the tangent line until we arrive at the point (x_1, y_1), where $x_1 = x_0 + h$ and $y_1 = y_0 + h \cdot F(x_0, y_0)$.

Then, using (x_1, y_1) as a new starting point, we can repeat the process to obtain a second point (x_2, y_2). This process can be repeated until we obtain an approximation for the desired x-value.

When using Euler's Method, note that we can obtain better approximations of the exact solution by choosing smaller and smaller step sizes.



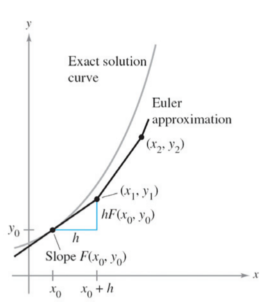
Euler's Method

Another way to think about Euler's Method is through the use of Δx and Δy.

The change in x between subsequent points, Δx, is simply the step size h.

The change in y between subsequent points, Δy, can be obtained by multiplying the slope at the starting point, $F(x_0, y_0)$, by Δx. Thus, $\Delta y = F(x_0, y_0) \cdot \Delta x$.

Add Δx and Δy to x_0 and y_0, respectively, to obtain x_1 and y_1. Repeat the process to obtain subsequent points.



$$x_1 = x_0 + h \qquad x_1 = x_0 + \Delta x$$
$$y_1 = y_0 + h \cdot F(x_0, y_0) \qquad y_1 = y_0 + F(x_0, y_0) \cdot \Delta y$$

Example: Euler's Method

Given that $y(0) = 1$ and $y' = 4xy$, what is the approximation for $y(2)$ if Euler's Method is used with a step size of 0.5, starting at $x = 0$? Show the work that leads to your answer.

Example: Euler's Method

Given that $y(0)=0$ and $y'=y+\cos(x)$, what is the approximation for $y(1)$ if Euler's Method is used with a step size of 0.2, starting at $x=0$? Show the work that leads to your answer.