§5.1a Slope Fields

General and Particular Solutions

Slope Fields

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Verify solutions to differential equations.
- Use slope fields to approximate solutions of differential equations.

General and Particular Solutions

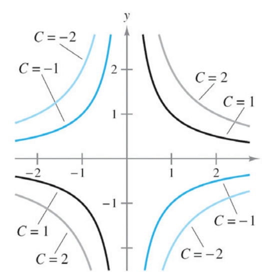
Recall that a **differential equation** in x and y is an equation that involves x, y, and derivatives of y. For example, $2xy' - 3y = 0$ is a differential equation.

A function $y = f(x)$ is called a **solution** of a differential equation if the equation is satisfied when y and its derivatives are replaced by $f(x)$ and its derivatives.

A set of solutions that differ only by one or more arbitrary constants is called the **general solution** of a differential equation.

The **order** of a differential equation is determined by the highest-order derivative in the equation. For instance, $y' = 4y$ is a first-order differential equation, while $y'' - y = 0$ is a second-order differential equation.

General and Particular Solutions



Geometrically, the general solution of a first-order differential equation represents a family of curves known as **solution curves**, one for each value assigned to the arbitrary constant.

For instance, we can verify that every function of the form $y = \frac{C}{x}$ is a solution of the differential equation $xy' + y = 0$.

The figure shows four of the solution curves corresponding to different values of C.

Particular solutions of a differential equation are obtained from **initial conditions** that give the values of the dependent variable or one of its derivatives for particular values of the independent variable.

Example: General and Particular Solutions

Verify the solution of the differential equation.

Solution: $y = \frac{2}{5}(e^{-4x} + e^{x})$ DE: $y'' + 4y' = 2e^{x}$

Example: General and Particular Solutions

Verify the particular solution of the differential equation.

Solution: $y = 6x - 4\sin(x) + 1$ DE: $y' = 6 - 4\cos(x)$ IC: $y(0) = 1$

Slope Fields

Solving a differential equation analytically can be difficult or even impossible. However, there is a graphical approach we can use to learn a lot about the solution of a differential equation.

Consider a differential equation of the form $y' = F(x, y)$, where $F(x, y)$ is some expression in x and y. At each point (x, y) in the xy-plane where F is defined, the differential equation determines the slope $y' = F(x, y)$ of the solution at that point.

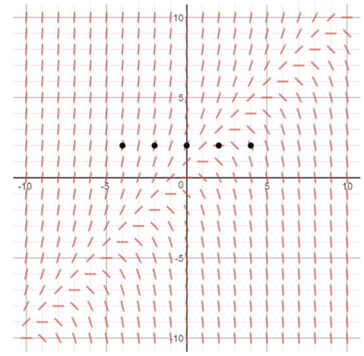
If we draw short line segments with slope $F(x, y)$ at selected points (x, y) in the domain of F, then these line segments form a **slope field**, or a *direction field*, for the differential equation $y' = F(x, y)$.

A slope field shows the general shape of all the solutions and can be helpful in getting a visual perspective of the directions of the solutions of a differential equation. A solution curve of a differential equation $y' = F(x, y)$ is simply a curve in the xy-plane whose tangent line at each point (x, y) has slope equal to $F(x, y)$.

Example: Slope Fields

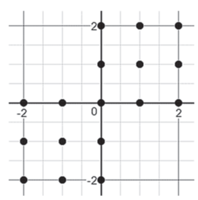
The slope field for the differential equation $\frac{dy}{dx} = y - x$ is given. Complete the table by determining the slopes (if possible) in the slope field at the given points.

x	-4	-2	0	2	4
y	2	2	2	2	2
y'					



Example: Slope Fields

Sketch a slope field for the differential equation $y' = \sqrt{xy}$ through the given points. Then sketch the solution curve that passes through the point (0, 0).



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