§4.4c The Net Change Theorem

Net Change Theorem

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Understand and use the Net Change Theorem.

Net Change Theorem

The Fundamental Theorem of Calculus states that if f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

But because $F'(x) = f(x)$, this statement can be rewritten as $\int_a^b F'(x)\,dx = F(b) - F(a)$.

The quantity $F(b) - F(a)$ represents the *net change of F* on the interval $[a, b]$.

Net Change Theorem

THEOREM THE NET CHANGE THEOREM

The definite integral of the rate of change of a quantity $F'(x)$ gives the total change, or **net change,** in that quantity on the interval $[a, b]$.

$$\int_a^b F'(x)\,dx = F(b) - F(a) \qquad \text{Net change of } F$$

Example: Net Change Theorem

Find the particular solution that satisfies the differential equation $f''(x) = 6x - 2$ and the initial condition $f(3) = 2$ and $f'(2) = 5$.

Example: Net Change Theorem

Given $f(1) = 0$ and $f'(x) = \frac{3}{x}$, find an equation of the line tangent to the graph of $f(x)$ at $x = 4$.

Example: Net Change Theorem

Let f be a twice-differentiable function with $f'(x) > 0$ and $f''(x) < 0$ for all x, such that $f(2) = 6$ and $f(3) = 11$. Explain why $11 < f(4) < 16$ must be true.

Example: Net Change Theorem

A container is being filled with water at a rate of $f(t) = \dfrac{30t}{t^3 + 4}$ cubic inches per second. At the same time, water leaks at a constant rate of $g(t) = 50$ cubic inches per second. At time $t = 0$, there are 1,000 cubic inches of water in the container.

Evaluate $\int_0^{20} (f(t) - g(t))\,dt$ using a graphing calculator. Using appropriate units, interpret the meaning of this integral in the context of this problem.

Example: Net Change Theorem

A particle moves along a vertical line so that its acceleration at time $t \geq 0$ is $a(t) = \sin(2t - 0.9)$. On the interval $0 \leq t \leq 3$, find the time at which the velocity of the particle is at a minimum. Justify your answer.

Net Change Theorem

Another way to illustrate the Net Change Theorem is to examine the velocity of a particle moving alongside a straight line, where $s(t)$ is the position at time t. Then its velocity is $v(t) = s'(t)$ and $\int_a^b v(t)\,dt = s(b) - s(a)$. This definite integral represents the net change in position, or **displacement**, of the particle.

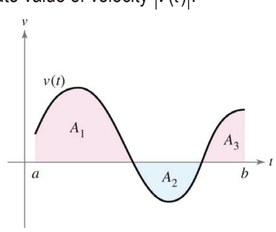
Net Change Theorem

When calculating the *total distance* traveled by the particle, we must consider the intervals where $v(t) < 0$ and the intervals where $v(t) > 0$. When $v(t) < 0$, the particle moves to the left (or down), and when $v(t) > 0$, the particle moves to the right (or up).

To calculate the total distance traveled, integrate the absolute value of velocity $|v(t)|$.

Given the graph of $v(t)$ in the diagram, where A_1, A_2, and A_3 are the areas of the respective regions:

The **displacement** of the particle on the interval $[a, b]$ is $\int_a^b v(t)\,dt = A_1 - A_2 + A_3$ and the **total distance** traveled by the particle on $[a, b]$ is $\int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$.



Example: Net Change Theorem

A particle is moving along a horizontal line so that its velocity at time $t \geq 0$ is $v(t) = \ln(t + 0.5)$.

(a) What is the displacement of the particle on the interval $0 \leq t \leq 4$?

(b) What is the total distance traveled by the particle on the interval $0 \leq t \leq 4$?