§4.4b The Second Fundamental Theorem of Calculus

The Second Fundamental Theorem of Calculus

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

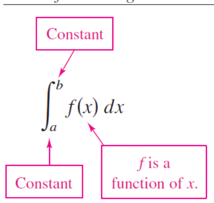
- Understand and use the Second Fundamental Theorem of Calculus.

The Second Fundamental Theorem of Calculus

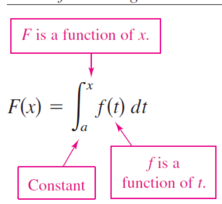
Earlier, we saw that the definite integral of f on the interval $[a, b]$ as defined using the constant b as the upper limit of integration and x as the variable of integration.

However, a slightly different situation may arise in which the variable x is used in the upper limit of integration. To avoid the confusion of using x in two different ways, t is temporarily used as the variable of integration.

The Definite Integral as a Number



The Definite Integral as a Function of x



The Second Fundamental Theorem of Calculus

Suppose $F(x) = \int_0^x \cos(t)\,dt$.

If we attempt to evaluate this definite integral using the Fundamental Theorem of Calculus:

$$F(x) = \int_0^x \cos(t)\,dt = \left[\sin(t)\right]_{t=0}^{t=x}$$
$$= \sin(x) - \sin(0) = \sin(x) - 0$$
$$= \sin(x)$$

Note that the derivative of F is the original integrand (with only the variable changed). That is:

$$\frac{d}{dx}[F(x)] = \frac{d}{dx}[\sin(x)] = \frac{d}{dx}\left[\int_0^x \cos(t)\,dt\right] = \cos(x)$$

This result is generated in the next theorem, called the **Second Fundamental Theorem of Calculus**.

The Second Fundamental Theorem of Calculus

THEOREM THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

Two things are necessary to utilize this theorem:

(1) The lower limit of integration must be a constant value.

(2) The upper limit of integration must be x.

The Second Fundamental Theorem of Calculus

Otherwise, we must use a more generalized version of this theorem.

Suppose $F(x) = \int_a^{g(x)} f(t)\,dt$. If we say that $u = g(x)$, then $F(x) = \int_a^u f(t)\,dt$.

Utilizing the Chain Rule, we get $F'(x) = f(u)\,u' = f(g(x)) \cdot g'(x)$.

Thus, a more generalized version of the Second Fundamental Theorem of Calculus states:

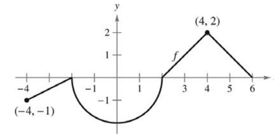
$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x)) \cdot g'(x)$$

Example: The Second Fundamental Theorem of Calculus

Find $F'(x)$ if $F(x)=\int_{\pi^2}^{x^6}\sqrt[3]{t}\,dt$.

Example: The Second Fundamental Theorem of Calculus

The graph of the function f shown consists of a semicircle and three line segments. Let g be the function given by $g(x)=\int_{-2}^{x}f(t)\,dt$.



(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

Example: The Second Fundamental Theorem of Calculus

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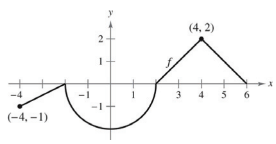
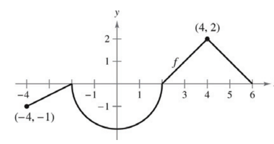


(b) Find all values of x in the open interval $(-4, 6)$ at which the graph of g is increasing and concave downward. Justify your answer.

Example: The Second Fundamental Theorem of Calculus

The graph of the function f shown consists of a semicircle and three line segments. Let g be the function given by $g(x)=\int_{-2}^{x}f(t)\,dt$.



(c) Find all values of x in the open interval $(-4, 6)$ at which g attains a relative minimum. Justify your answer.

Example: The Second Fundamental Theorem of Calculus

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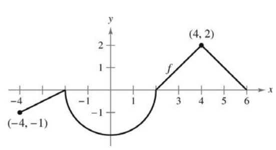
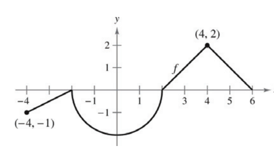


(d) Find the absolute minimum value of g on the closed interval $[-4, 6]$. Justify your answer.

Example: The Second Fundamental Theorem of Calculus

The graph of the function f shown consists of a semicircle and three line segments. Let g be the function given by $g(x)=\int_{-2}^{x}f(t)\,dt$.



(e) Find all values of x in the open interval $(-4, 6)$ at which the graph of g has a point of inflection. Justify your answer.