§3.6 Optimization Problems

The First Derivative Test for Absolute Extrema

Applied Minimum and Maximum Problems

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to…

- Apply the First Derivative Test for Absolute Extrema to find absolute extrema of a function.
- Solve applied minimum and maximum problems.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.

2.2A Use derivatives to analyze properties of a function.

2.3C Solve problems involving related rates, optimization, rectilinear motion, *(BC: and planar motion)*.

The First Derivative Test for Absolute Extrema

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x) < 0$ for all $x < c$ on I, and $f'(x) > 0$ for all $x > c$ on I, then f has an *absolute minimum* at $(c, f(c))$.
2. If $f'(x) > 0$ for all $x < c$ on I, and $f'(x) < 0$ for all $x > c$ on I, then f has an *absolute maximum* at $(c, f(c))$.

Text adapted from *Paul's Online Math Notes* by Paul Dawkins. http://tutorial.math.lamar.edu/index.aspx

Example: The First Derivative Test for Absolute Extrema

Given the function $f(x) = \dfrac{x}{1+x^2}$ with $f'(x) = \dfrac{(1+x)(1-x)}{(1+x^2)^2}$, find the x-coordinate of the absolute maximum value of f for $x \geq 0$. Justify your answer.

Example: The First Derivative Test for Absolute Extrema

For $0 < t < 3$, a particle moves along the x-axis with position given by $x(t) = t^3 - 9t^2 + 24t$. Find the time t at which the particle is farthest to the right. Justify your answer.

Applied Minimum and Maximum Problems

GUIDELINES FOR SOLVING APPLIED MINIMUM AND MAXIMUM PROBLEMS

1. Identify all *given* quantities and all quantities *to be determined*. If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the back cover.)
3. Reduce the primary equation to one having a *single independent variable*. This may involve the use of **secondary equations** relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
5. Determine the desired maximum or minimum value using the First Derivative Test for Absolute Extrema.

Example: Applied Minimum and Maximum Problems

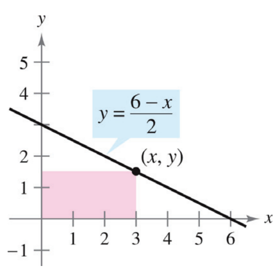
Find two positive numbers such that the sum of the first number and twice the second number is 108 and the product is a maximum.

Example: Applied Minimum and Maximum Problems

Find the point on the graph of $f(x) = -x^2$ that is closest to the point $(16, -1/2)$.

Example: Applied Minimum and Maximum Problems

A rectangle is bound by the x- and y-axes and the graph of $y = \dfrac{6-x}{2}$. What length and width should the rectangle have so that its area is a maximum?



Example: Applied Minimum and Maximum Problems

A rancher has 600 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?