

§3.5 A Summary of Curve Sketching

Analyzing the Graph of a Function

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Analyze and sketch the graph of a function.
- Identify and describe the graph of f , given the graph of f' .
- Identify and describe the graph of f' , given the graph of f .

Learning Objectives: Students will be able to...

- 1.1A Express limits symbolically using correct notation, and interpret limits expressed symbolically.
- 1.1C Determine limits of functions.
- 1.1D Deduce and interpret behavior of functions using limits.
- 1.2A Analyze functions for intervals of continuity and points of discontinuity.
- 2.1C Calculate derivatives.
- 2.1D Determine higher order derivatives.
- 2.2A Use derivatives to analyze properties of a function.

Analyzing the Graph of a Function

GUIDELINES FOR ANALYZING THE GRAPH OF A FUNCTION

1. Determine the domain and range of the function.
2. Determine the intercepts, asymptotes, and symmetry of the graph.
3. Locate the x -values for which $f'(x)$ and $f''(x)$ either are zero or do not exist. Use the results to determine relative extrema and points of inflection.

Analyzing the Graph of a Function

1. State all x -intercepts and y -intercepts.
2. State all vertical and horizontal asymptotes. Justify the horizontal asymptotes using limits.
3. State all intervals on which the function is continuous.
4. State all intervals on which the function is increasing/decreasing. Justify your answer.
5. State all intervals on which the graph of the function is concave upward/downward. Justify your answer.
6. State all points (with both x - and y -values) where the function has relative extrema. Justify your answer.
7. State all points (with both x - and y -values) where the graph of the function has points of inflection. Justify your answer.
8. If any of the above characteristics do not exist, state and show that there are none.

Example: Analyzing the Graph of a Function

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function (cont.)

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function (cont.)

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function (cont.)

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function (cont.)

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function (cont.)

Analyze and sketch a graph of $y = \frac{x^2}{2(x^2 - 1)}$.

Example: Analyzing the Graph of a Function
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

Example: Analyzing the Graph of a Function (cont.)
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

Example: Analyzing the Graph of a Function (cont.)
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

Example: Analyzing the Graph of a Function (cont.)
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

Example: Analyzing the Graph of a Function (cont.)
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

Example: Analyzing the Graph of a Function (cont.)
Analyze and sketch a graph of $y = x\sqrt{x+2}$.

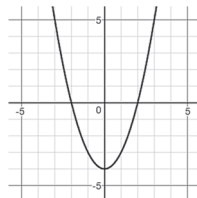
Example: Analyzing the Graph of a Function
 Analyze and sketch a graph of $y = x^3 - 6x^2 + 9x$.

Example: Analyzing the Graph of a Function (cont.)
 Analyze and sketch a graph of $y = x^3 - 6x^2 + 9x$.

Example: Analyzing the Graph of a Function (cont.)
 Analyze and sketch a graph of $y = x^3 - 6x^2 + 9x$.

Example: Analyzing the Graph of a Function (cont.)
 Analyze and sketch a graph of $y = x^3 - 6x^2 + 9x$.

Example: Analyzing the Graph of a Function
 The graph of f' , the derivative of f , is shown in the figure.
 Describe the graph of f .



Example: Analyzing the Graph of a Function
 The graph of f is shown in the figure. Describe the graph of f' , the derivative of f .

