§3.4b The Second Derivative Test

The Second Derivative Test

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to…

- Apply the Second Derivative Test to find relative extrema of a function.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.
2.1D Determine higher order derivatives.
2.2A Use derivatives to analyze properties of a function.

The Second Derivative Test

THEOREM SECOND DERIVATIVE TEST

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

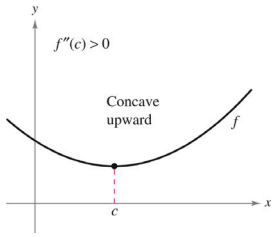
1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.

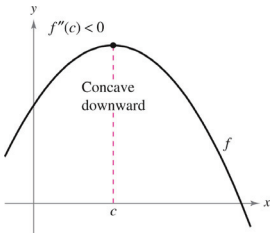
The Second Derivative Test only works for critical numbers where $f'(c) = 0$. For critical numbers where f' does not exist, you must use the First Derivative Test.

The Second Derivative Test

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.



☺ **happy** people are **positive**



☹ **sad** people are **negative**

Example: The Second Derivative Test

Given the function $f(x) = -x^3 + 3x^2 - 2$, identify the x-coordinate of all relative extrema. Use the Second Derivative Test where applicable. Justify your answer.

Example: The Second Derivative Test

Given the function $f(x) = x^{1/2} + x^{-1/2}$, identify the x-coordinate of all relative extrema. Use the Second Derivative Test where applicable. Justify your answer.

Example: The Second Derivative Test

Given the function $f(x) = x^{-1} + x^{-2}$, identify the x-coordinate of all relative extrema. Use the Second Derivative Test where applicable. Justify your answer.

Example: The Second Derivative Test

Given the function $f(x) = x + 2\cos(x)$, $0 < x < 2\pi$, identify the x-coordinate of all relative extrema. Use the Second Derivative Test where applicable. Justify your answer.

Example: The Second Derivative Test

Consider the differential equation $\frac{dy}{dx} = x + y^2$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(-1) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = -1$? Justify your answer.

Example: The Second Derivative Test (cont.)

Consider the differential equation $\frac{dy}{dx} = x + y^2$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(-1) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = -1$? Justify your answer.