

§3.3b The First Derivative Test
The First Derivative Test

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Apply the First Derivative Test to find relative extrema of a function.

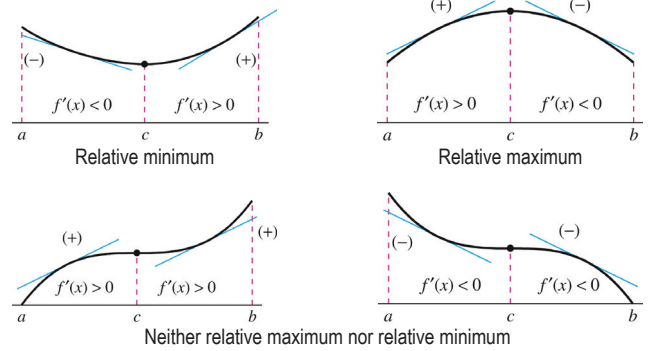
The First Derivative Test

THEOREM THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c . If f is differentiable on the interval, except possibly at c , then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c , then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c , then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c , then $f(c)$ is neither a relative minimum nor a relative maximum.

The First Derivative Test

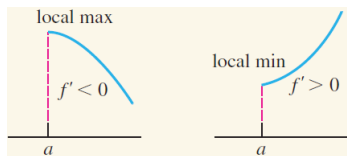


The First Derivative Test

At a left endpoint a :

If $f'(x) < 0$ for $x > a$, then f has a relative maximum at $(a, f(a))$.

If $f'(x) > 0$ for $x > a$, then f has a relative minimum at $(a, f(a))$.



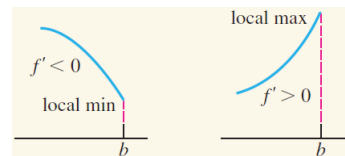
Text/Images from *Calculus: Graphical, Numerical, Algebraic, 3rd ed.* by Finney, Demana, Waits, & Kennedy. © 2007 Pearson Education, Inc.

The First Derivative Test

At a right endpoint b :

If $f'(x) < 0$ for $x < b$, then f has a relative minimum at $(b, f(b))$.

If $f'(x) > 0$ for $x < b$, then f has a relative maximum at $(b, f(b))$.



Text/Images from *Calculus: Graphical, Numerical, Algebraic, 3rd ed.* by Finney, Demana, Waits, & Kennedy. © 2007 Pearson Education, Inc.

Example: The First Derivative Test

Given the function $f(x) = x^3 - 6x^2 + 15$, identify the x -coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

Given the function $f(x) = (4 - x^2)^{1/2}$, identify the x -coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

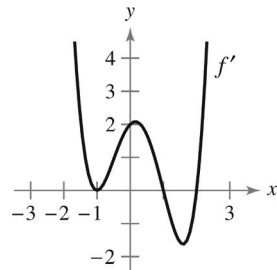
Given the function $f(x) = x^{-1} + 4x^{-2}$, identify the x -coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

Given the function $f(x) = \sqrt{3} \sin(x) + \cos(x)$, $0 < x < 2\pi$, identify the x -coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

The graph of f' , the derivative of f , is shown. For $-3 < x < 3$, identify the x -coordinates of all relative extrema. Justify your answer.



Example: The First Derivative Test

For $0 \leq t \leq 3$, a particle moves along the x -axis with position given by $x(t) = t^3 - 9t^2 + 24t$. Find all times t at which the particle changes direction. Justify your answer.