§3.3b The First Derivative Test

The First Derivative Test

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Apply the First Derivative Test to find relative extrema of a function.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.

2.2A Use derivatives to analyze properties of a function.

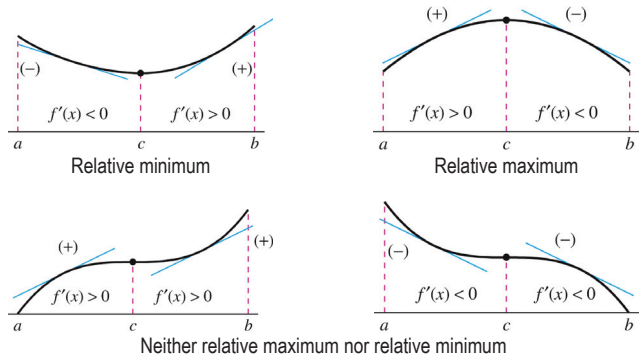
The First Derivative Test

THEOREM THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.

The First Derivative Test

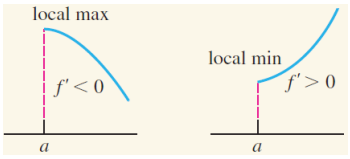


The First Derivative Test

At a left endpoint a:

If $f'(x) < 0$ for $x > a$, then f has a relative maximum at $(a, f(a))$.

If $f'(x) > 0$ for $x > a$, then f has a relative minimum at $(a, f(a))$.



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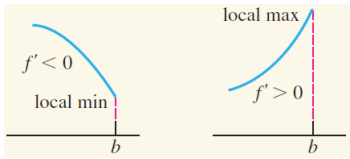
The First Derivative Test

At a right endpoint b:

If $f'(x) < 0$ for $x < b$, then f has a relative minimum at $(b, f(b))$.

If $f'(x) > 0$ for $x < b$, then f has a relative maximum at $(b, f(b))$.



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Example: The First Derivative Test

Given the function $f(x) = x^3 - 6x^2 + 15$, identify the x-coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

Given the function $f(x) = (4 - x^2)^{1/2}$, identify the x-coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

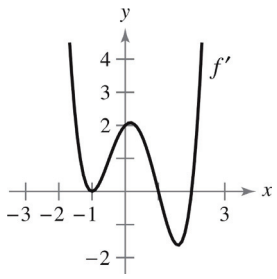
Given the function $f(x) = x^{-1} + 4x^{-2}$, identify the x-coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

Given the function $f(x) = \sqrt{3}\sin(x) + \cos(x)$, $0 < x < 2\pi$, identify the x-coordinates of all relative extrema. Justify your answer.

Example: The First Derivative Test

The graph of f', the derivative of f, is shown. For $-3 < x < 3$, identify the x-coordinates of all relative extrema. Justify your answer.



Example: The First Derivative Test

For $0 \le t \le 3$, a particle moves along the x-axis with position given by $x(t) = t^3 - 9t^2 + 24t$. Find all times t at which the particle changes direction. Justify your answer.