§3.3a **Increasing and Decreasing Functions**

Increasing and Decreasing Functions

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Determine intervals on which a function is increasing or decreasing.
- Determine intervals on which a particle moves in a particular direction.
- Explain the meaning of a derivative at a point value in the context of the problem.

Increasing and Decreasing Functions

DEFINITIONS OF INCREASING AND DECREASING FUNCTIONS

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function is increasing when, *as x moves to the right*, its graph moves up.

A function is decreasing when, as x *moves to the right*, its graph moves down.

Increasing and Decreasing Functions

THEOREM TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

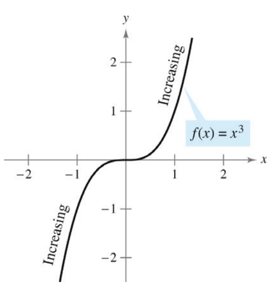
The conclusions in the first two cases are valid even when $f'(x) = 0$ at a finite number of x-values in $[a, b]$.

Increasing and Decreasing Functions

A function is **strictly monotonic** on an interval when it is either increasing on the entire interval or decreasing on the entire interval.

For instance, the function $f(x) = x^3$ is strictly monotonic on the entire real number line because it is increasing on the entire real number line, as shown in the figure.

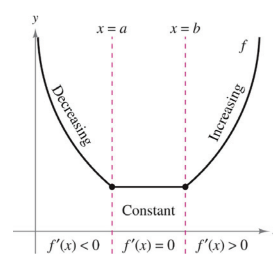
$$f'(x) = 3x^2 > 0 \text{ for all } x \neq 0$$



Increasing and Decreasing Functions

Let f be continuous on the interval $[a, b]$. To find the intervals on which f is increasing or decreasing, use the following steps.

1. Locate the critical numbers of f in $[a, b]$ and use these numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Use the aforementioned theorem to determine whether f is increasing or decreasing on each interval.



These guidelines are also valid when the interval $[a, b]$ is replaced by an interval of the form $(-\infty, b]$, $[a, \infty)$, or $(-\infty, \infty)$.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x) = x^3 - 6x^2 + 15$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x) = (4 - x^2)^{1/2}$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

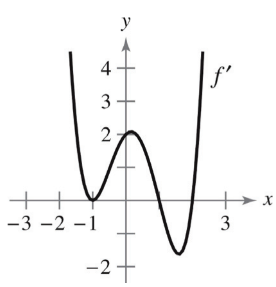
Find the interval(s) on which the function $f(x) = x^{-1} + 4x^{-2}$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x) = \sqrt{3}\sin(x) + \cos(x)$, $0 < x < 2\pi$, is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

The graph of f', the derivative of f, is shown. For $-3 < x < 3$, find the interval(s) on which the function $f(x)$ is increasing or decreasing. Justify your answer.



Example: Increasing and Decreasing Functions

For $0 \le t \le 3$, a particle moves along the x-axis with position given by $x(t) = t^3 - 9t^2 + 24t$.

(a) Find all times t during which the particle is moving to the left. Justify your answer.

(b) Find all times t at which the particle is at rest. Justify your answer.

Example: Increasing and Decreasing Functions

A tank of water initially contains 50 gallons of water at time $t = 0$. During the time interval $0 \le t \le 4$ hours, water is pumped into the tank at the rate $B(t) = \sqrt{t}$ gallons per hour. During the same time interval, water is removed from the tank at the rate $C(t) = \sqrt[3]{t}$ gallons per hour. Let $D(t) = B(t) - C(t)$.

Using a graphing calculator, find $D'(2)$. Using correct units, explain the meaning of that value in the context of this problem.