## §3.3a Increasing and Decreasing Functions

Increasing and Decreasing Functions

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
*Calculus, AP Edition, 9th ed.* by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

**Learning Goals: Students will be able to…**

- Determine intervals on which a function is increasing or decreasing.
- Determine intervals on which a particle moves in a particular direction.
- Explain the meaning of a derivative at a point value in the context of the problem.

**Learning Objectives: Students will be able to…**

2.1C Calculate derivatives.
2.2A Use derivatives to analyze properties of a function.
2.3A Interpret the meaning of a derivative within a problem.
2.3C Solve problems involving related rates, optimization, rectilinear motion, *(BC: and planar motion)*.
2.3D Solve problems involving rates of change in applied contexts.

### Increasing and Decreasing Functions

**DEFINITIONS OF INCREASING AND DECREASING FUNCTIONS**

A function $f$ is **increasing** on an interval if for any two numbers $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function $f$ is **decreasing** on an interval if for any two numbers $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function is increasing when, *as x moves to the right*, its graph moves up.

A function is decreasing when, as $x$ *moves to the right*, its graph moves down.

### Increasing and Decreasing Functions

**THEOREM TEST FOR INCREASING AND DECREASING FUNCTIONS**

Let $f$ be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$.

**1.** If $f'(x) > 0$ for all $x$ in $(a, b)$, then $f$ is increasing on $[a, b]$.
**2.** If $f'(x) < 0$ for all $x$ in $(a, b)$, then $f$ is decreasing on $[a, b]$.
**3.** If $f'(x) = 0$ for all $x$ in $(a, b)$, then $f$ is constant on $[a, b]$.

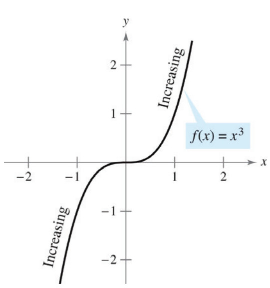
The conclusions in the first two cases are valid even when $f'(x) = 0$ at a finite number of $x$-values in $[a, b]$.

### Increasing and Decreasing Functions

A function is **strictly monotonic** on an interval when it is either increasing on the entire interval or decreasing on the entire interval.

For instance, the function $f(x) = x^3$ is strictly monotonic on the entire real number line because it is increasing on the entire real number line, as shown in the figure.

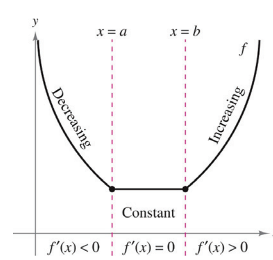
$$f'(x) = 3x^2 > 0 \text{ for all } x \neq 0$$




### Increasing and Decreasing Functions

Let $f$ be continuous on the interval $[a, b]$. To find the intervals on which $f$ is increasing or decreasing, use the following steps.

1. Locate the critical numbers of $f$ in $[a, b]$ and use these numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Use the aforementioned theorem to determine whether $f$ is increasing or decreasing on each interval.




These guidelines are also valid when the interval $[a, b]$ is replaced by an interval of the form $(-\infty, b]$, $[a, \infty)$, or $(-\infty, \infty)$.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x)=x^3-6x^2+15$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x)=(4-x^2)^{1/2}$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

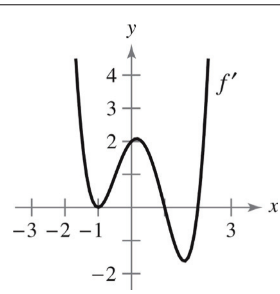
Find the interval(s) on which the function $f(x)=x^{-1}+4x^{-2}$ is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

Find the interval(s) on which the function $f(x)=\sqrt{3}\,\sin(x)+\cos(x)$, $0<x<2\pi$, is increasing or decreasing. Justify your answer.

Example: Increasing and Decreasing Functions

The graph of $f'$, the derivative of $f$, is shown. For $-3<x<3$, find the interval(s) on which the function $f(x)$ is increasing or decreasing. Justify your answer.




Example: Increasing and Decreasing Functions

For $0\le t\le 3$, a particle moves along the $x$-axis with position given by $x(t)=t^3-9t^2+24t$.

(a) Find all times $t$ during which the particle is moving to the left. Justify your answer.

(b) Find all times $t$ at which the particle is at rest. Justify your answer.

Example: Increasing and Decreasing Functions

A tank of water initially contains 50 gallons of water at time $t = 0$. During the time interval $0 \le t \le 4$ hours, water is pumped into the tank at the rate $B(t) = \sqrt{t}$ gallons per hour. During the same time interval, water is removed from the tank at the rate $C(t) = \sqrt[3]{t}$ gallons per hour. Let $D(t) = B(t) - C(t)$.

Using a graphing calculator, find $D'(2)$. Using correct units, explain the meaning of that value in the context of this problem.