

§3.2 Rolle's Theorem and the Mean Value Theorem

Rolle's Theorem

The Mean Value Theorem

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Understand and use Rolle's Theorem.
- Understand and use the Mean Value Theorem.

Learning Objectives: Students will be able to...

- 1.2B Determine the applicability of important calculus theorems using continuity.
 2.1A Identify the derivative of a function as the limit of a difference quotient.
 2.1B Estimate derivatives.
 2.1C Calculate derivatives.
 2.4A Apply the Mean Value Theorem to describe the behavior of a function over an interval.

Rolle's Theorem

THEOREM **ROLLE'S THEOREM**

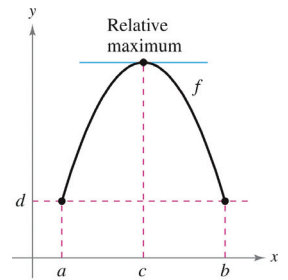
Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.

Rolle's Theorem

If a function f is continuous on $[a, b]$ and differentiable on (a, b) , and if $f(a) = f(b)$, then there must be at least one x -value between a and b at which the graph of f has a horizontal tangent.



Example: Rolle's Theorem

Determine whether Rolle's Theorem can be applied to $f(x) = x^2 - 5x + 5$ on the interval $[1, 4]$. If so, find all guaranteed values of c . If not, explain why not.

The Mean Value Theorem

THEOREM **THE MEAN VALUE THEOREM**

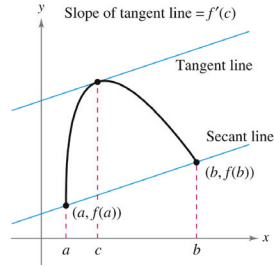
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The Mean Value Theorem

Geometrically, the MVT guarantees the existence of a tangent line that is parallel to the secant line through the points $(a, f(a))$ and $(b, f(b))$.

In terms of rate of change, the MVT implies that there must be a point in the open interval (a, b) at which the instantaneous rate of change is equal to the average rate of change over the interval $[a, b]$.



Example: The Mean Value Theorem

Determine whether the Mean Value Theorem can be applied to $f(x) = x^4 - 12x$ on the interval $[0, 2]$. If so, find all guaranteed values of c . If not, explain why not.

Example: The Mean Value Theorem

Determine whether the Mean Value Theorem can be applied to $f(x) = 5 - \frac{4}{x}$ on the interval $[1, 4]$. If so, find all guaranteed values of c . If not, explain why not.

Example: The Mean Value Theorem

Determine whether the Mean Value Theorem can be applied to $f(x) = 5 - \frac{4}{x}$ on the interval $[-1, 1]$. If so, find all guaranteed values of c . If not, explain why not.

Example: The Mean Value Theorem

Two stationary patrol cars are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 miles per hour. The truck passes the second patrol car four minutes later. Prove that the truck must have exceeded the speed limit of 55 miles per hour at some time during the 4 minutes.