§2.7 Related Rates

Finding Related Rates

Problem Solving with Related Rates

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Find a related rate.
- Use related rates to solve real-life problems.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.

2.3C Solve problems involving related rates, optimization, rectilinear motion, *(BC: and planar motion)*.

2.3D Solve problems involving rates of change in applied contexts.

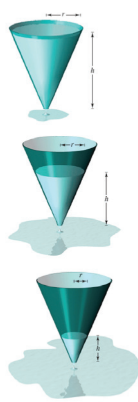
Finding Related Rates

We have seen how the Chain Rule can be used to find dy/dx implicitly.

Another important use of the Chain Rule is to find the rates of change of two or more related variables that are changing with respect to *time*.

For example, when water is drained out of a conical tank, the volume V, the radius r, and the height h of the water level are all functions of time t.

We know that these variables are related by the equation $V = \frac{1}{3}\pi r^2 h$.



Finding Related Rates

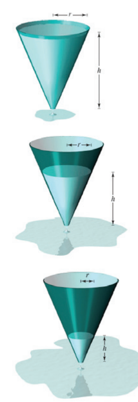
We can differentiate implicitly with respect to t to obtain the **related-rate** equation

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]$$

$$V'(t) = \frac{1}{3}\pi r^2 \cdot h'(t) + \frac{2}{3}\pi r \cdot r'(t) \cdot h$$

$$= \frac{1}{3}\pi\left(r^2\, h'(t) + 2rh\, r'(t)\right)$$

From this equation, we can see that the rate of change of V is related to the rates of change of both h and r.



Problem Solving with Related Rates

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time* t.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

Problem Solving with Related Rates

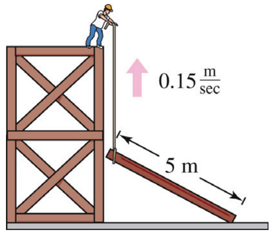
Verbal Statement	Mathematical Model
The velocity of a car after traveling for 1 hour is 50 miles per hour.	$x =$ distance traveled $\frac{dx}{dt} = 50$ when $t = 1$
Water is being pumped into a swimming pool at a rate of 10 cubic meters per hour.	$V =$ volume of water in pool $\frac{dV}{dt} = 10\ \text{m}^3/\text{hr}$
A gear is revolving at a rate of 25 revolutions per minute (1 revolution $= 2\pi$ rad).	$\theta =$ angle of revolution $\frac{d\theta}{dt} = 25(2\pi)$ rad/min

Example: Problem Solving with Related Rates

A point is moving along the graph of $xy = 4$ at the rate $dx/dt = 10$. Find dy/dt when $x = 8$.

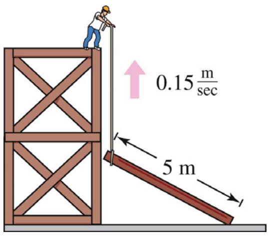
Example: Problem Solving with Related Rates

A construction worker pulls a 5-meter plank up the side of a building by means of a rope tied to one end of the plank. Assume the opposite end of the plank follows a path perpendicular to the wall of the building, and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 3 meters from the wall of the building?



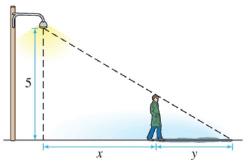
Example: Problem Solving with Related Rates (cont.)

A construction worker pulls a 5-meter plank up the side of a building by means of a rope tied to one end of the plank. Assume the opposite end of the plank follows a path perpendicular to the wall of the building, and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 3 meters from the wall of the building?



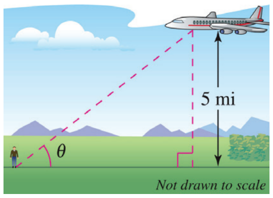
Example: Problem Solving with Related Rates

A man of height 1.8 meters walks away from a 5-meter lamppost at a speed of 1.2 meter per second. Find the rate at which his shadow is increasing in length.



Example: Problem Solving with Related Rates

An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation θ is changing when the angle is $\theta = \pi/6$.



Example: Problem Solving with Related Rates

A right cylindrical water tank with a diameter of 3 feet and a height of 6 feet is being drained. At what rate is the water level of the tank changing when the volume of water in the tank is dropping at a rate of 0.75π cubic feet per minute?