§2.5 Implicit Differentiation

Implicit and Explicit Functions

Implicit Differentiation

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to…

- Distinguish between functions written in implicit form and explicit form.
- Use implicit differentiation to find the derivative of a function.

Implicit and Explicit Functions

Up until this lesson, most functions have been expressed in **explicit form**.

For example, in the equation $y = 3x^2 - 5$, the variable y is explicitly written as a function of x.

Some functions, however, are only implied by an equation. For instance, the function $y = 1/x$ is defined **implicitly** by the equation $xy = 1$.

To find dy/dx for this equation, we can write y explicitly as a function of x and then differentiate.

Implicit Form	*Explicit Form*	*Derivative*
$xy = 1$	$y = \frac{1}{x} = x^{-1}$	$\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$

For some equations, such as $x^2 - 2y^3 + 4y = 2$, it is difficult to express y as a function of x explicitly. To find dy/dx, we can use **implicit differentiation**.

Implicit Differentiation

To understand how to find dy/dx implicitly, we must realize that the differentiation is taking place *with respect to* x.

This means that when you differentiate terms involving x alone, you can differentiate as usual.

However, when you differentiate terms involving y, you must apply the Chain Rule, because you are assuming that y is defined implicitly as a differentiable function of x.

Implicit Differentiation

GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides of the equation *with respect to* x.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

Note: You can switch "left" and "right" in these guidelines.

The resulting equation, which has dy/dx on one side and an expression in terms of x and y on the other, is sometimes called a **differential equation**.

Example: Implicit Differentiation

Find dy/dx by implicit differentiation: $x^2y + xy^2 = -2$

Example: Implicit Differentiation

Find an equation of the line tangent to the graph of $4x^3 + \ln(y^2) + 2y = 2x$ at the point $(-1, 1)$.

Example: Implicit Differentiation

Find d^2y/dx^2 implicitly in terms of x and y: $x^2 + y^2 = 4$

Example: Implicit Differentiation

If $x^2y - 4x = 5$, what is the value of d^2y/dx^2 at the point $(-1, 1)$?