§2.4a The Chain Rule

The Chain Rule

The General Power Rule

Transcendental Functions and the Chain Rule

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Find the derivative of a composite function using the Chain Rule.
- Find the derivative of a function using the General Power Rule.
- Simplify the derivative of a function using algebra.
- Find the derivative of a trigonometric function using the Chain Rule.

Learning Objectives: Students will be able to…

2.1C Calculate derivatives.

2.1D Determine higher order derivatives.

The Chain Rule

THEOREM THE CHAIN RULE

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

The Chain Rule

When applying the Chain Rule, it is helpful to think of the composite function $f \circ g$ as having two parts—an inner part and an outer part.

Outer function

$$y = f(g(x)) = f(u)$$

Inner function

The derivative of $y = f(u)$ is the derivative of the outer function (at the inner function u) *times* the derivative of the inner function: $y' = f'(u) \cdot u'$

The General Power Rule

THEOREM THE GENERAL POWER RULE

If $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[u^n] = nu^{n-1}u'.$$

Example: The General Power Rule

Find the derivative of $y = \dfrac{2}{(5-x^2)^3}$.

Example: The General Power Rule

Find the derivative of $f(x) = x\sqrt{x^4 + 1}$.

Transcendental Functions and the Chain Rule

$$\frac{d}{dx}[\sin u] = (\cos u)\,u' \qquad \frac{d}{dx}[\cos u] = -(\sin u)\,u'$$

$$\frac{d}{dx}[\tan u] = (\sec^2 u)\,u' \qquad \frac{d}{dx}[\cot u] = -(\csc^2 u)\,u'$$

$$\frac{d}{dx}[\sec u] = (\sec u \tan u)\,u' \qquad \frac{d}{dx}[\csc u] = -(\csc u \cot u)\,u'$$

Example: Transcendental Functions and the Chain Rule

Find the second derivative of $f(\theta) = \sin(4\theta^2)$.

Example: Transcendental Functions and the Chain Rule

Find the derivative of $y = \dfrac{\tan(4x)}{x^3}$.

Example: Transcendental Functions and the Chain Rule

Find the derivative of $g(t) = \sec^3(5t)$.