

§1.5 Infinite Limits

Infinite Limits
 Vertical Asymptotes
 Properties of Infinite Limits

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to...

- Determine infinite limits from the left and from the right.
- Find the vertical asymptotes of the graph of a function.

Infinite Limits

Given $f(x) = \frac{3}{x-2}$, find the limits $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ numerically and graphically.

Explain your reasoning.

x approaches 2 from the left.
x approaches 2 from the right.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	-30	-300	-3000	?	3000	300	30

f(x) decreases without bound.
f(x) increases without bound.

Infinite Limits

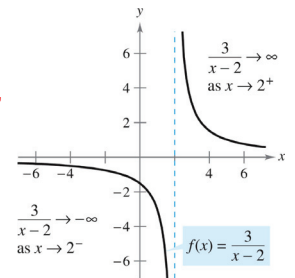
Given $f(x) = \frac{3}{x-2}$, find the limits $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ numerically and graphically.

Explain your reasoning.

$\lim_{x \rightarrow 2^-} f(x) = -\infty$ (DNE). As x approaches 2 from the left, $f(x)$ decreases without bound.

$\lim_{x \rightarrow 2^+} f(x) = +\infty$ (DNE). As x approaches 2 from the right, $f(x)$ increases without bound.

A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an **infinite limit**.



Example: Infinite Limits

Find the limit (if it exists) analytically.

$$\lim_{x \rightarrow 2^-} \frac{x}{x-2}$$

Example: Infinite Limits

Find the limit (if it exists) analytically.

$$\lim_{x \rightarrow (\pi/2)^+} \frac{-2}{\cos(x)}$$

Vertical Asymptotes

DEFINITION OF VERTICAL ASYMPTOTE

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f .

Note: Only one of the one-sided infinite limits is required to show that there is a vertical asymptote.

Example: Vertical Asymptotes

Find the vertical asymptotes (if any) of the graph of the function $f(x) = \frac{x^2 - 6x - 7}{x^2 - 1}$.
Justify your answer using a table of values and one-sided limits. Explain your reasoning.

Properties of Infinite Limits

THEOREM PROPERTIES OF INFINITE LIMITS

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x \rightarrow c} f(x) = \infty \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = L.$$

1. Sum or difference: $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \infty$

2. Product: $\lim_{x \rightarrow c} [f(x)g(x)] = \infty, \quad L > 0$

$$\lim_{x \rightarrow c} [f(x)g(x)] = -\infty, \quad L < 0$$

3. Quotient: $\lim_{x \rightarrow c} \frac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.