§1.5 Infinite Limits

Infinite Limits

Vertical Asymptotes

Properties of Infinite Limits

Learning Goals: Students will be able to…

- Determine infinite limits from the left and from the right.
- Find the vertical asymptotes of the graph of a function.

Learning Objectives: Students will be able to…

1.1A Express limits symbolically using correct notation, and interpret limits expressed symbolically.

1.1B Estimate limits of functions.

1.1C Determine limits of functions.

1.1D Deduce and interpret behavior of functions using limits.

1.2A Analyze functions for intervals of continuity or points of discontinuity.

Infinite Limits

Given $f(x)=\dfrac{3}{x-2}$, find the limits $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$ numerically and graphically.

Explain your reasoning.

x approaches 2 from the left. | x approaches 2 from the right.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	-30	-300	-3000	?	3000	300	30

$f(x)$ decreases without bound. | $f(x)$ increases without bound.

Infinite Limits

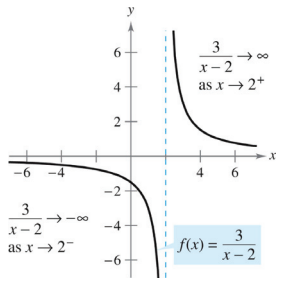
Given $f(x)=\dfrac{3}{x-2}$, find the limits $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$ numerically and graphically.

Explain your reasoning.

$\lim\limits_{x\to 2^-} f(x)=-\infty$ (DNE). As x approaches 2 from the left, $f(x)$ decreases without bound.

$\lim\limits_{x\to 2^+} f(x)=+\infty$ (DNE). As x approaches 2 from the right, $f(x)$ increases without bound.

A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an **infinite limit**.



Example: Infinite Limits

Find the limit (if it exists) analytically.

$$\lim_{x\to 2^+} \frac{x}{x-2}$$

Example: Infinite Limits

Find the limit (if it exists) analytically.

$$\lim_{x\to (\pi/2)^+} \frac{-2}{\cos(x)}$$

Vertical Asymptotes

DEFINITION OF VERTICAL ASYMPTOTE

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

Note: Only one of the one-sided infinite limits is required to show that there is a vertical asymptote.

Example: Vertical Asymptotes

Find the vertical asymptotes (if any) of the graph of the function $f(x) = \dfrac{x^2 - 6x - 7}{x^2 - 1}$.

Justify your answer using a table of values and one-sided limits. Explain your reasoning.

Properties of Infinite Limits

THEOREM PROPERTIES OF INFINITE LIMITS

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L.$$

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x \to c} [f(x)g(x)] = -\infty, \quad L < 0$
3. Quotient: $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.