

§1.2 Finding Limits Graphically and Numerically

- An Introduction to Limits
- Estimating a Limit Numerically
- Estimating a Limit Graphically
- Limits That Fail to Exist

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
Calculus, AP Edition, 9th ed. by Larson & Edwards. © 2010 Brooks/Cole, Cengage Learning.

Learning Goals: Students will be able to...

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.

An Introduction to Limits

Given: $f(x) = \frac{x^3 - 1}{x - 1}, x \neq 1$

What happens to $f(x)$ as x gets closer to 1?

x approaches 1 from the left.
x approaches 1 from the right.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	2.710	2.970	2.997	?	3.003	3.030	3.310

f(x) approaches 3.
f(x) approaches 3.

An Introduction to Limits

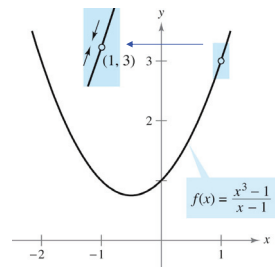
Given: $f(x) = \frac{x^3 - 1}{x - 1}, x \neq 1$

What happens to $f(x)$ as x gets closer to 1?

Although x cannot equal 1, we can move arbitrarily close to 1, and as a result $f(x)$ moves arbitrarily close to 3.

Using limit notation, we can write: $\lim_{x \rightarrow 1} f(x) = 3$

This is read as "the limit of $f(x)$ as x approaches 1 is 3."



An Introduction to Limits

This discussion leads to an informal definition of limit.

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, then the **limit** of $f(x)$, as x approaches c , is L .

This limit is written as $\lim_{x \rightarrow c} f(x) = L$.

Example: Estimating a Limit Numerically

Create a table of values for the function $f(x) = \frac{x+3}{x^2 + 7x + 12}$ and use the result to estimate the limit $\lim_{x \rightarrow -3} f(x)$. Explain your reasoning.

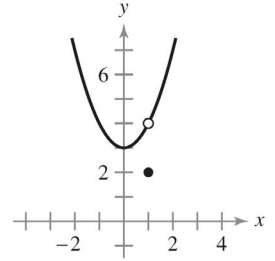
Example: Estimating a Limit Numerically

Create a table of values for the function $f(x) = \begin{cases} 2x - 1 & \text{for } x < 2 \\ x^2 - 1 & \text{for } x > 2 \end{cases}$ and use the result to estimate the limit $\lim_{x \rightarrow 2} f(x)$. Explain your reasoning.

Example: Estimating a Limit Graphically

The graph of the function $f(x)$ is shown in the figure.

- (a) Find the value of $f(1)$.
- (b) Find the value of $\lim_{x \rightarrow 1} f(x)$. Explain your reasoning.



Limits That Fail to Exist

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c .
3. $f(x)$ oscillates between two fixed values as x approaches c .

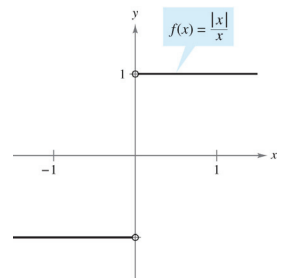
We say that these limits “do not exist” (DNE).

1. Jump
2. Vertical asymptote
3. Rapid oscillation with sine or cosine

Example: Limits That Fail to Exist

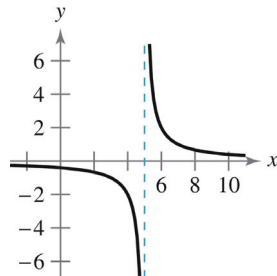
The graph of the function $f(x) = \frac{|x|}{x}$ is shown in the figure.

Find the value of $\lim_{x \rightarrow 0} f(x)$ numerically and graphically. Explain your reasoning.



Example: Limits That Fail to Exist

The graph of the function $f(x) = \frac{2}{x-5}$ is shown in the figure. Find the value of $\lim_{x \rightarrow 5} f(x)$ numerically and graphically. Explain your reasoning.



Example: Limits That Fail to Exist

The graph of the function $f(x) = \sin\left(\frac{1}{x}\right)$ is shown in the figure. Find the value of $\lim_{x \rightarrow 0} f(x)$ graphically. Explain your reasoning.

