§1.2 Finding Limits Graphically and Numerically

An Introduction to Limits
Estimating a Limit Numerically
Estimating a Limit Graphically
Limits That Fail to Exist

Notes based on: *Calculus for AP* by Larson & Battaglia. © 2017 Cengage Learning.
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Learning Goals: Students will be able to…

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.

Learning Objectives: Students will be able to…

1.1A Express limits symbolically using correct notation, and interpret limits expressed symbolically.
1.1B Estimate limits of functions.
1.1D Deduce and interpret behavior of functions using limits.

An Introduction to Limits

Given: $f(x)=\dfrac{x^3-1}{x-1},\ x\neq 1$

What happens to $f(x)$ as x gets closer to 1?

x approaches 1 from the left. → ← x approaches 1 from the right.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	2.710	2.970	2.997	?	3.003	3.030	3.310

$f(x)$ approaches 3. → ← $f(x)$ approaches 3.

An Introduction to Limits

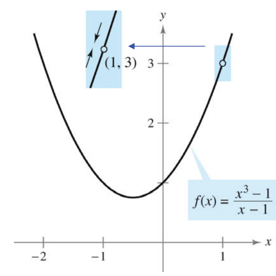
Given: $f(x)=\dfrac{x^3-1}{x-1},\ x\neq 1$

What happens to $f(x)$ as x gets closer to 1?

Although x cannot equal 1, we can move arbitrarily close to 1, and as a result $f(x)$ moves arbitrarily close to 3.

Using limit notation, we can write: $\lim\limits_{x\to 1} f(x)=3$

This is read as "the limit of $f(x)$ as x approaches 1 is 3."



An Introduction to Limits

This discussion leads to an informal definition of limit.

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, then the **limit** of $f(x)$, as x approaches c, is L.

This limit is written as $\lim\limits_{x\to c} f(x)=L$.

Example: Estimating a Limit Numerically

Create a table of values for the function $f(x)=\dfrac{x+3}{x^2+7x+12}$ and use the result to estimate the limit $\lim\limits_{x\to -3} f(x)$. Explain your reasoning.

Example: Estimating a Limit Numerically

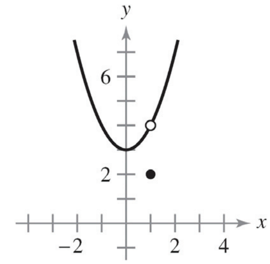
Create a table of values for the function $f(x)=\begin{cases} 2x-1 & \text{for } x<2 \\ x^2-1 & \text{for } x>2 \end{cases}$ and use the result to estimate the limit $\lim_{x\to 2} f(x)$. Explain your reasoning.

Example: Estimating a Limit Graphically

The graph of the function $f(x)$ is shown in the figure.

(a) Find the value of $f(1)$.

(b) Find the value of $\lim_{x\to 1} f(x)$. Explain your reasoning.



Limits That Fail to Exist

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

We say that these limits "do not exist" (DNE).

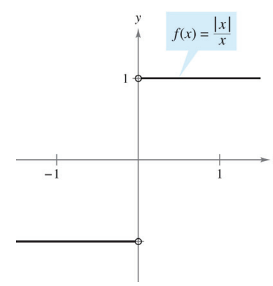
1. Jump
2. Vertical asymptote
3. Rapid oscillation with sine or cosine

Example: Limits That Fail to Exist

The graph of the function $f(x)=\frac{|x|}{x}$ is shown in the figure.

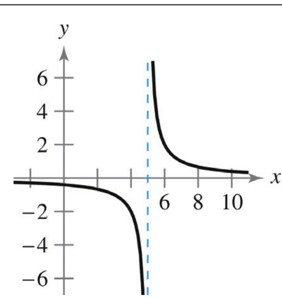
Find the value of $\lim_{x\to 0} f(x)$ numerically and graphically.

Explain your reasoning.



Example: Limits That Fail to Exist

The graph of the function $f(x)=\frac{2}{x-5}$ is shown in the figure. Find the value of $\lim_{x\to 5} f(x)$ numerically and graphically. Explain your reasoning.



Example: Limits That Fail to Exist

The graph of the function $f(x)=\sin\left(\frac{1}{x}\right)$ is shown in the figure. Find the value of $\lim_{x\to 0} f(x)$ graphically. Explain your reasoning.